Artificial Intelligence

Lecturer 7 – Part II: Decision Tree & Reinforcement Learning

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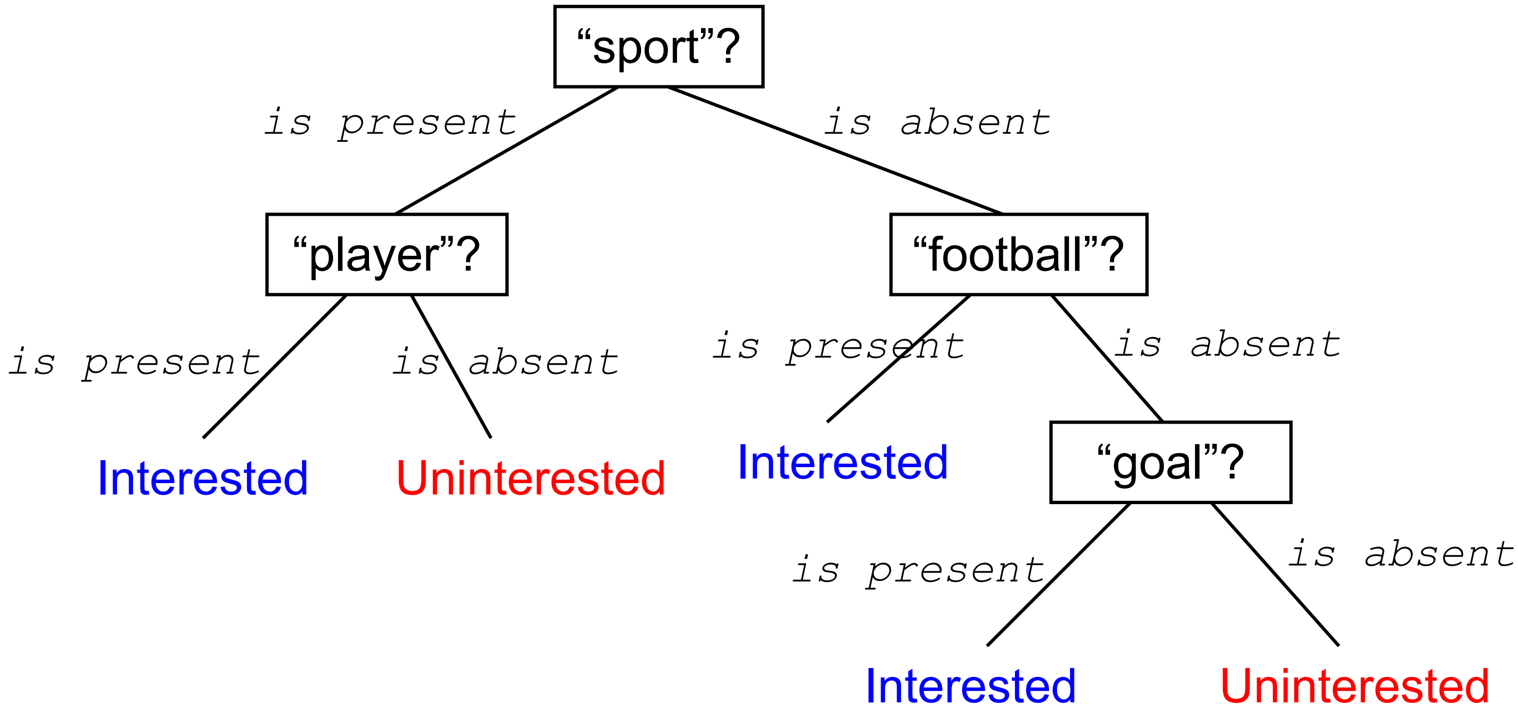
Decision tree – Introduction

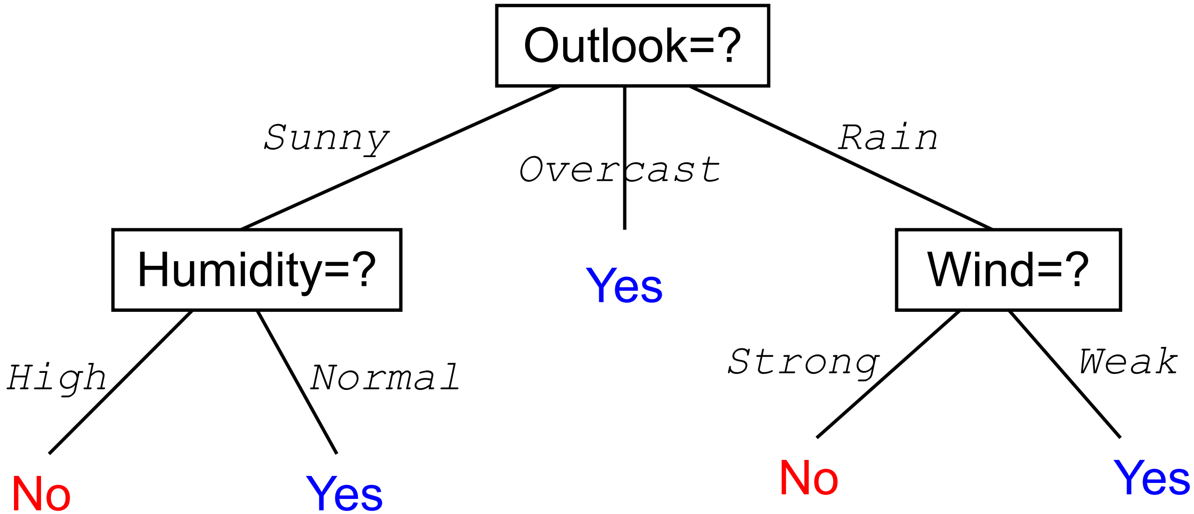
- Decision tree (DT) learning
  - To approximate a ***discrete-valued target function***
  - The target function is represented by a decision tree
- A DT can be represented (interpreted) as a set of IF-THEN rules (i.e., easy to read and understand)
- Capable of learning disjunctive expressions
- DT learning is robust to noisy data
- One of the most widely used methods for inductive inference
- Successfully applied to a range of real-world applications

Example of a DT: Which documents are of my interest?



- (…,"sport",…,"player",…) → Interested
- (…,"goal",…) → Interested
- (…,"sport",…) → Uninterested

Example of a DT: Does a person play tennis?



- (Outlook=`Overcast`, Temperature=`Hot`, Humidity=`High`, Wind=`Weak`) → Yes
- (Outlook=`Rain`, Temperature=`Mild`, Humidity=`High`, Wind=`Strong`) → No
- (Outlook=`Sunny`, Temperature=`Hot`, Humidity=`High`, Wind=`Strong`) → No

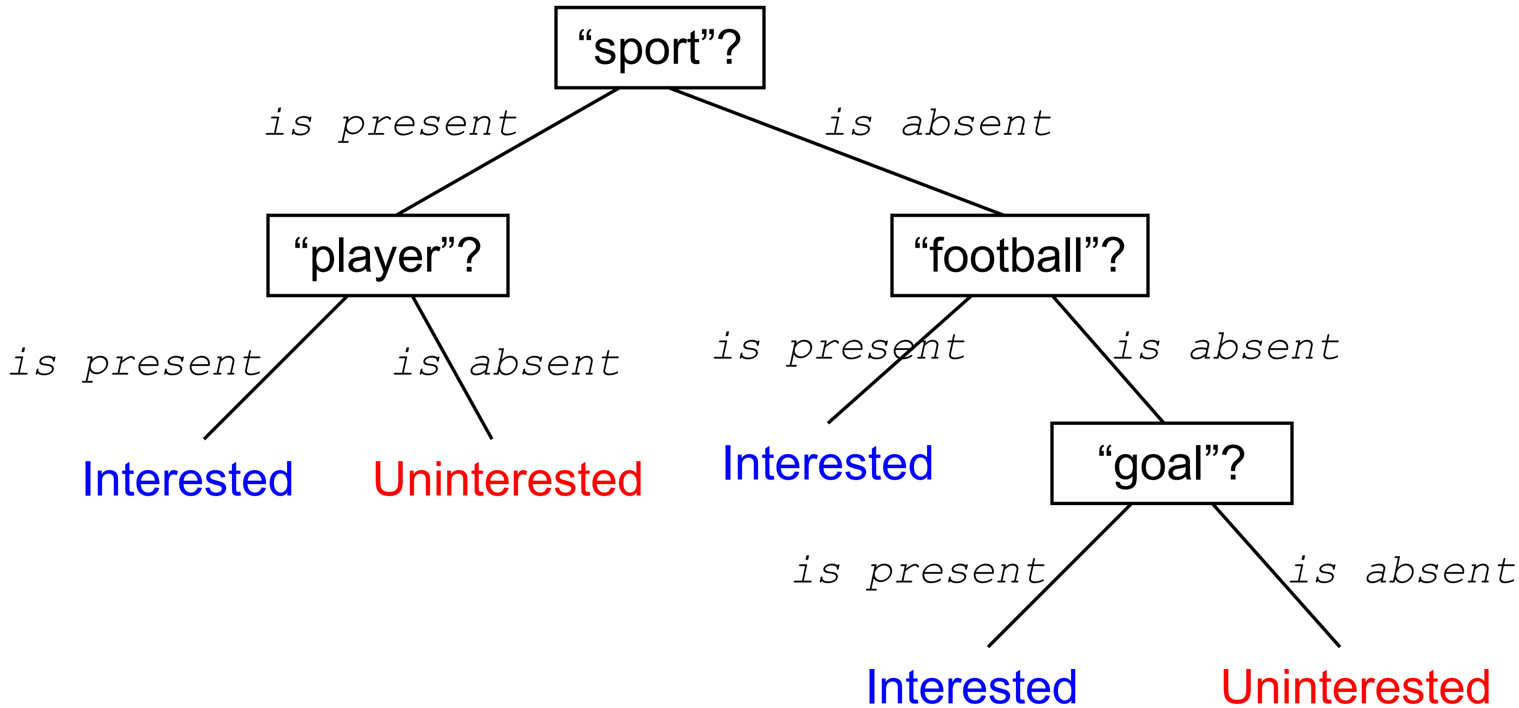
Decision tree – Representation (1)

- Each *internal node* represents an *attribute to be tested* by instances
- Each *branch* from a node corresponds to *a possible value of the attribute* associated with that node
- Each *leaf node* represents a *classification* (e.g., a class label)
- A learned DT classifies an instance by sorting it down the tree, from the root to some leaf node
  - → The classification associated with the leaf node is used for the instance

Decision tree – Representation (2)

- A DT represents a disjunction of conjunctions of constraints on the attribute values of instances
- Each path from the root to a leaf corresponds to a conjunction of attribute tests
- The tree itself is a disjunction of these conjunctions
- Examples
  - → Let's consider the two previous example DTs…

Which documents are of my interest?

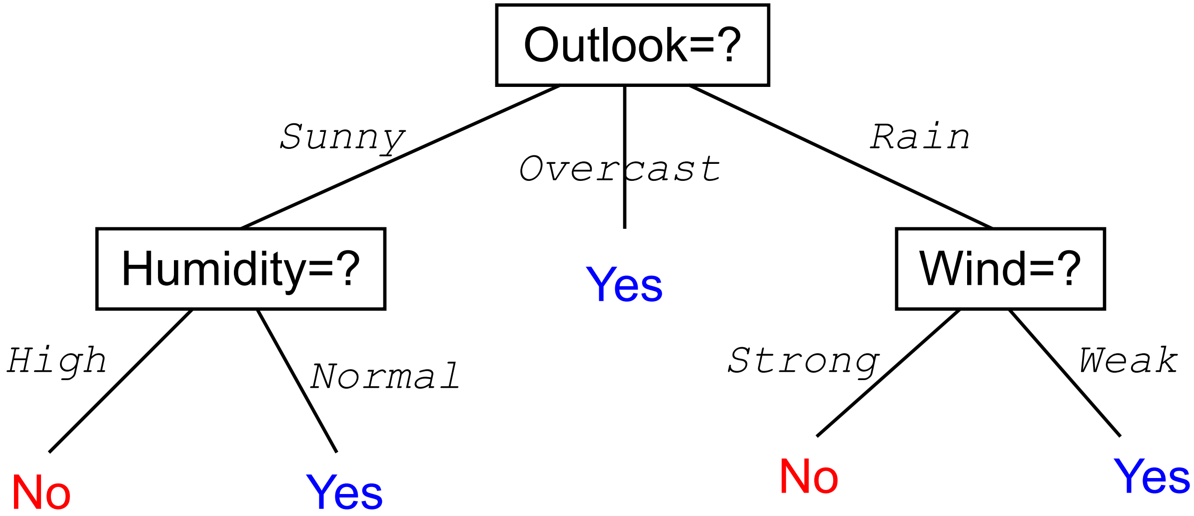


[("sport" `is present`) $\wedge$ ("player" `is present`)] $\vee$

[("sport" `is absent`) $\wedge$ ("football" `is present`)] $\vee$

[("sport" `is absent`) $\wedge$ ("football" `is absent`) $\wedge$ ("goal" `is present`)]

Does a person play tennis?



[(Outlook=`Sunny`) $\wedge$ (Humidity=`Normal`)] $\vee$

(Outlook=`Overcast`) $\vee$

[(Outlook=`Rain`) $\wedge$ (Wind=`Weak`)]

Decision tree learning – ID3 algorithm

ID3_alg(`Training_Set`, `Class_Labels`, `Attributes`)

Create a node `Root` for the tree

If all instances in `Training_Set` have the same class label `c`, Return the tree of the single-node `Root` associated with class label `c`

If the set `Attributes` is empty, Return the tree of the single-node `Root` associated with class label ≡ **Majority_Class_Label**(`Training_Set`)

`A` ← The attribute in `Attributes` that "best" classifies `Training_Set`

The test attribute for node `Root` ← `A`

For each possible value `v` of attribute `A`

Add a new tree branch under `Root`, corresponding to the test: "value of attribute `A` is `v`"

Compute $\mathtt{Training\_Set}_v$ = {instance `x` | `x` ⊆ `Training_Set`, x_A=v}

If ($\mathtt{Training\_Set}_v$ is empty) Then

Create a leaf node with class label ≡ **Majority_Class_Label**(`Training_Set`)

Attach the leaf node to the new branch

Else Attach to the new branch the sub-tree **ID3_alg**($\mathtt{Training\_Set}_v$, `Class_Labels`, {`Attributes` \ `A`})

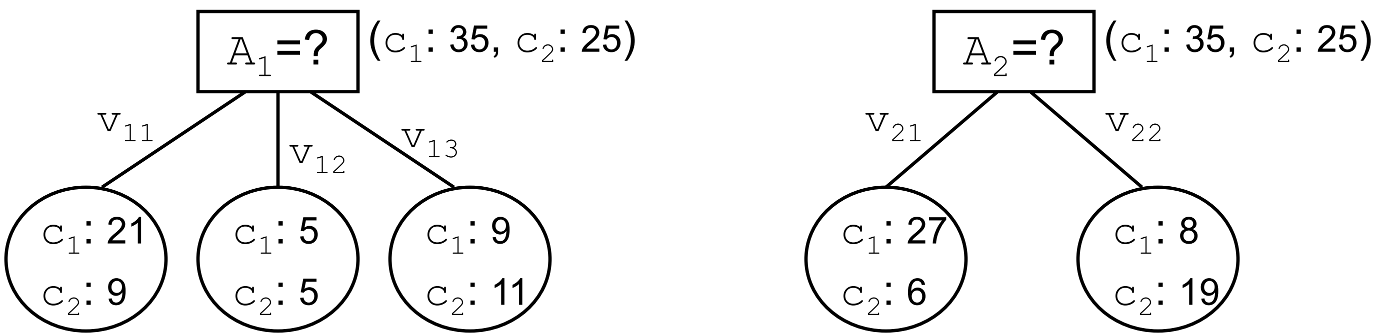
Return `Root`

ID3 algorithm – Intuitive idea

- Perform a greedy search through the space of possible DTs
- Construct (i.e., learn) a DT in a top-down fashion, starting from its root node
- At each node, the test attribute is the one (of the candidate attributes) that best classifies the training instances associated with the node
- A descendant (sub-tree) of the node is created for each possible value of the test attribute, and the training instances are sorted to the appropriate descendant node
- Every attribute can appear at most once along any path of the tree
- The tree growing process continues
  - Until the (learned) DT perfectly classifies the training instances, or
  - Until all the attributes have been used

Selection of the test attribute

- A very important task in DT learning: at each node, how to choose the test attribute?
- To select the attribute that is most useful for classifying the training instances associated with the node
- How to measure an attribute's capability of separating the training instances according to their target classification
  - → Use a statistical measure – *Information Gain*
- Example: A two-class (c_1, c_2) classification problem
  - → Which attribute, A_1 or A_2, should be chosen to be the test attribute?



Entropy

- A commonly used measure in the Information Theory field
- To measure the impurity (inhomogeneity) of a set of instances
- The entropy of a set S relative to a c-class classification

$$Entropy(S) = \sum_{i=1}^{c} - p_i . \log_2 p_i$$

where p_i is the proportion of instances in S belonging to class i, and
  $0.\log_2 0=0$ (convention)

- The entropy of a set S relative to a two-class classification

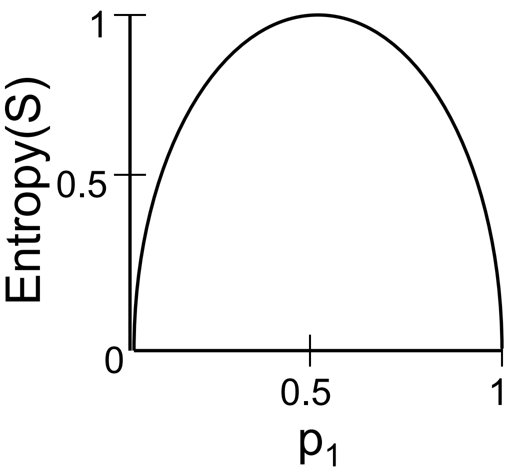
$$\mathrm{Entropy(S)} = -p_1.\log_2 p_1 - p_2.\log_2 p_2$$

- Interpretation of entropy (in the Information Theory field)
  → The entropy of S specifies the expected number of bits needed to encode class of a member randomly drawn out of S
    - Optical length code assigns $-\log_2 p$ bits to message having probability p
    - The expected number of bits needed to encode a class: $p.\log_2 p$

Entropy – Two-class example

- S contains 14 instances, where 9 belongs to class c_1 and 5 to class c_2
- The entropy of S relative to the two-class classification:

$$\text{Entropy}(S) = -(9/14).\log_2(9/14) - (5/14).\log_2(5/14) \approx 0.94$$



- Entropy =0, if all the instances belong to the same class (either c_1 or c_2)
  →Need 0 bit for encoding (no message need be sent)
- Entropy =1, if the set contains equal numbers of c_1 and c_2 instances
  → Need 1 bit per message for encoding (whether c_1 or c_2)
- Entropy = some value in (0,1), if the set contains unequal numbers of c_1 and c_2 instances
  → Need on average <1 bit per message for encoding

Information gain

- **Information gain** of an attribute relative to a set of instances is
  - the expected reduction in entropy
  - caused by partitioning the instances according to the attribute
- Information gain of attribute `A` relative to set `S`

$$Gain(S, A) = Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

where `Values(A)` is the set of possible values of attribute `A`, and

$S_v = \{x \mid x \in S, x_A = v\}$

- In the above formula, the second term is the expected value of the entropy after `S` is partitioned by the values of attribute `A`
- Interpretation of `Gain(S,A)`: The number of bits saved (reduced) for encoding class of a randomly drawn member of `S`, by knowing the value of attribute `A`

Training set - Example

Let us consider the following dataset (of a person)
s:

Day	Outlook	Temperature	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

[Mitchell, 1997]

Information gain – Example

- What is the information gain of attribute `Wind` relative to the training set `S` – `Gain(S,Wind)`?
- Attribute `Wind` have two possible values: `Weak` and `Strong`
- `S` = {9 positive and 5 negative instances}
- S_{weak} = {6 pos. and 2 neg. instances having `Wind=Weak`}
- S_{strong} = { 3 pos. and 3 neg. instances having `Wind=Strong`}

$$Gain(S,Wind) = Entropy(S) - \sum_{v \in \{Weak,Strong\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

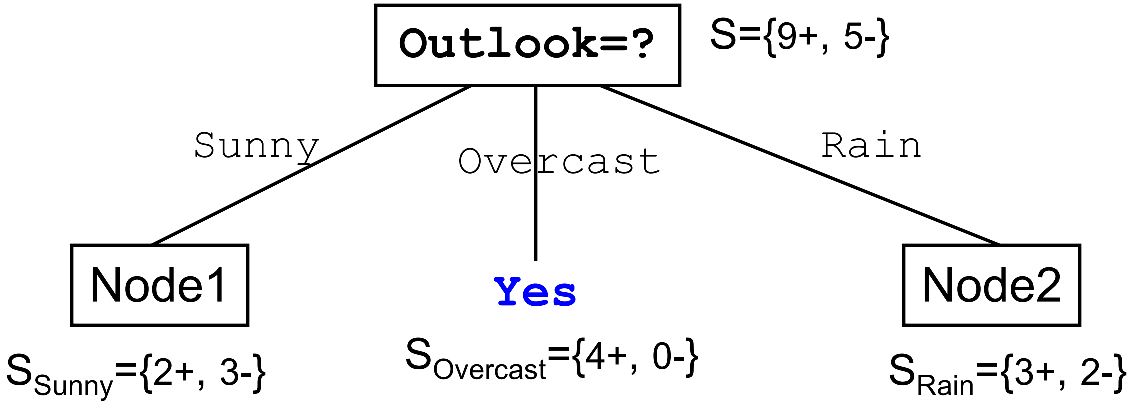
$$= Entropy(S) - (8/14).Entropy(S_{Weak}) - (6/14).Entropy(S_{Strong})$$

$$= 0.94 - (8/14).(0.81) - (6/14).(1) = 0.048$$

Decision tree learning – Example (1)

- At the root node, which attribute of {`Outlook`, `Temperature`, `Humidity`, `Wind`} should be the test attribute?
  - `Gain(S,` **`Outlook`**`) = ... =` **`0.246`** ← The highest IG value
  - `Gain(S, Temperature) = ... = 0.029`
  - `Gain(S, Humidity) = ... = 0.151`
  - `Gain(S, Wind) = ... = 0.048`

→So, `Outlook` is chosen as the test attribute for the root node!



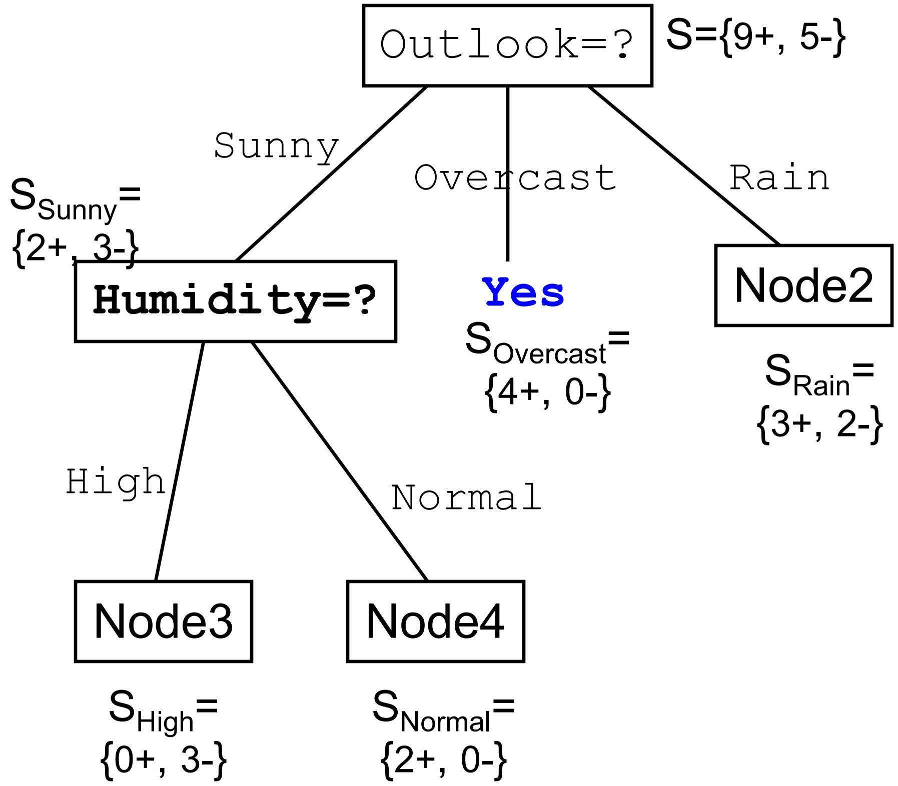
Decision tree learning – Example (2)

- At `Node1`, which attribute of {`Temperature`, `Humidity`, `Wind`} should be the test attribute?

  Note! Attribute `Outlook` is excluded, since it has been used by `Node1`'s parent (i.e., the root node)

  - Gain(S_{Sunny}, `Temperature`) =...= 0.57
  - Gain(S_{Sunny}, **`Humidity`**) = ... = **0.97**
  - Gain(S_{Sunny}, `Wind`) = ... = 0.019

→So, `Humidity` is chosen as the test attribute for `Node1`!



DT learning – Hypothesis space search (1)

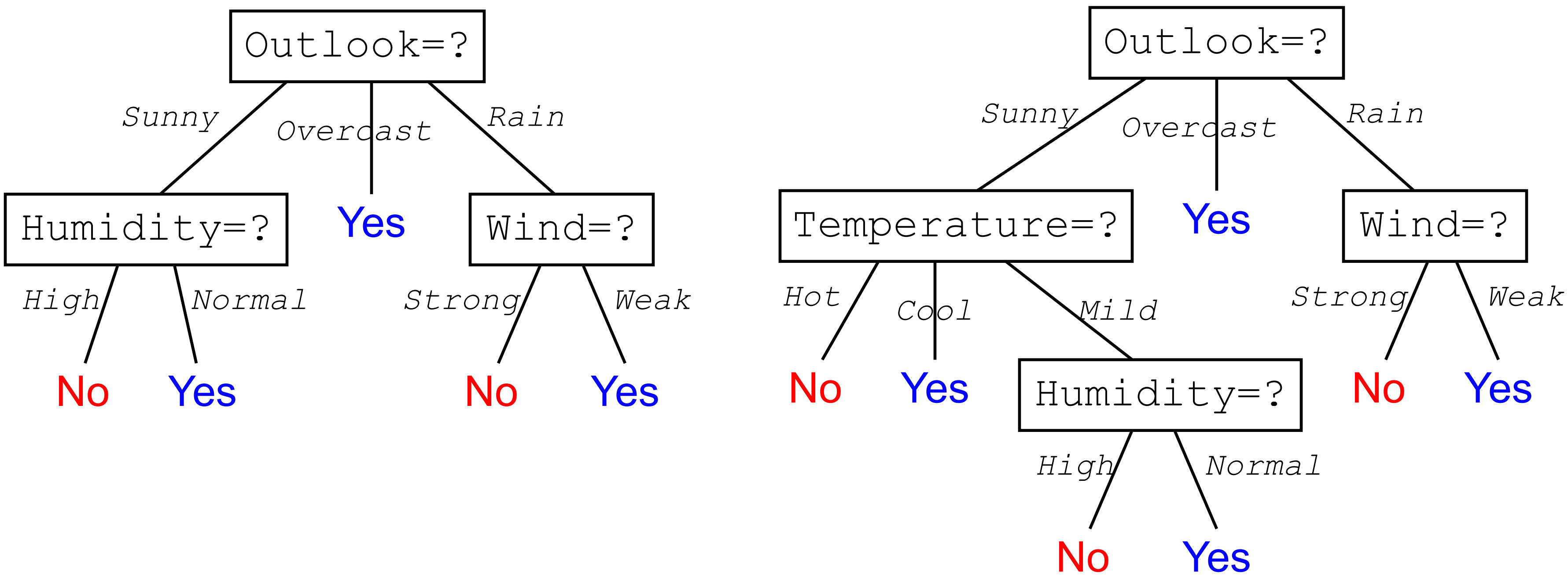
- Induction of Decision Trees (ID3) – Quinlan (1986)
- ID3 searches in a space of hypotheses (i.e., of possible DTs) for one that fits the training instances
- ID3 performs a simple-to-complex, hill-climbing search, beginning with the empty tree
- The hill-climbing search is guided by an evaluation metric – the information gain measure
- ID3 searches only one (rather than all possible) DT consistent with the training instances

DT learning – Hypothesis space search (2)

- ID3 does not performs backtracking in its search
  - → Guaranteed to converge to a locally (but not the globally) optimal solution
  - → Once an attribute is selected as the test for a node, ID3 never backtracks to reconsider this choice
- At each step in the search, ID3 uses a statistical measure of all the instances (i.e., information gain) to refine its current hypothesis
  - → The resulting search is much less sensitive to errors in individual training instances

Inductive bias in DT learning (1)

- Both the two DTs below are consistent with the given training dataset
- So, which one is preferred (i.e., selected) by the ID3 algorithm?



Inductive bias in DT learning (2)

- Given a set of training instances, there may be many DTs consistent with these training instances
- So, which of these candidate DTs should be chosen?
- ID3 chooses the first acceptable DT it encounters in its simple-to-complex, hill-climbing search
  →Recall that ID3 searches incompletely through the hypothesis space (i.e., without backtracking)
- ID3's search strategy
  - Select in favor of shorter trees over longer ones
  - Select trees that place the attributes with highest information gain closest to the root node

Issues in DT learning

- **Over-fitting the training data**
  - Overfitting: production of an analysis that corresponds too closely or exactly to a particular set of data, and may therefore fail to fit additional data or predict future observations reliably
- Handling continuous-valued (i.e., real-valued) attributes
- Choosing appropriate measures for attribute selection
- Handling training data with missing attribute values
- Handling attributes with differing costs

→ An extension of the ID3 algorithm with the above mentioned issues resolved results in the C4.5 algorithm

REINFORCEMENT LEARNING

Reinforcement Learning (RL)

- RL is ML method that optimizes the reward
  - A class of tasks
  - A process of trial-and-error learning
    - Good actions are "rewarded"
    - Bad actions are "punished"

Features of RL

- Learning from numerical rewards
- Interaction with the task; sequences of states, actions and rewards
- Uncertainty and non-deterministic worlds
- Delayed consequences
- The explore/exploit dilemma
- The whole problem of goal-directed learning

Points of view

- From the point of view of agents
  - RL is a process of trial-and-error learning
  - How much reward will I get if I do this action?
- From the point of view of trainers
  - RL is training by rewards and punishments
  - Train computers like we train animals

Applications of RL

- Robot
- Animal training
- Scheduling
- Games
- Control systems
- …

Supervised Learning vs. Reinforcement Learning

- Supervised learning
  - Teacher: Is this an AI course or a Math course?
  - Learner: Math
  - Teacher: No, AI
  - …
  - Teacher: Is this an AI course or a Math course?
  - Learner : AI
  - Teacher : Yes
- Reinforcement learning
  - World: You are in state 9. Choose action A or B
  - Learner: A
  - World: Your reward is 100
  - …
  - World: You are in state 15. Choose action C or D
  - Learner: D
  - World : Your reward is 50

Examples

- Chess
  - Win +1, loose -1
- Elevator dispatching
  - Reward based on mean squared time for elevator to arrive (optimization problem)
- Channel allocation for cellular phones
  - Lower rewards the more calls are blocked

Policy, Reward and Goal

- Policy
  - defines the agent's behaviour at a given time
  - maps from perceptions to actions
  - can be defined by: look-up table, neural net, search algorithm...
  - may be stochastic
- Reward Function
  - defines the goal(s) in an RL problem
  - maps from states, state-action pairs, or state-action-successor state, triplets to a numerical reward
  - goal of the agent is to maximise the total reward in the long run
  - the policy is altered to achieve this goal

Reward and Return

- The reward function indicates how good things are right now
- But the agent wants to maximize reward in the long-term i.e.. over many time steps
- We refer to long-term (multi-step) reward as return

$$R_t = r_{t+1} + r_{t+2} + \ldots + r_T$$

where

- T is the last time step of the world

Discounted Return

- The geometrically discounted model of return

$$R_t = r_{t+1} + \gamma r_{t+2} + \ldots + \gamma^T r_T$$
$$0 \le \gamma \le 1$$

  - γ is called discount rate, used to
    - Bound the infinite sum
    - Favor earlier rewards, in other words to give preference to shorter paths

Optimal Policies

- An RL agent adapts its policy in order to increase return
- A policy p_1 is at least as good as a policy p_2 if its expected return is at least as great in each possible initial state
- An optimal policy p is at least as good as any other policy

Policy Adaptation Methods

- Value function-based methods
  - Learn a value function for the policy
  - Generate a new policy from the value function
  - Q-learning, Dynamic Programming

Value Functions

- A value function maps each state to an estimate of return under a policy
- An action-value function maps from state-action pairs to estimates of return
- Learning a value function is referred to as the “prediction” problem or ‘policy evaluation’ in the Dynamic Programming literature

Q-learning

- Learns action-values *Q(s,a)* rather than state-values *V(s)*
- Action-values learning
  - *Q(s,a)* = value of doing action *a* in state *s*

$$Q(s,a) = R(s,a) + \gamma \max_{a'} Q(T(s,a),a')$$

- Q-learning improves action-values iteratively until it converges

Q-learning Algorithm

Algorithm Q {

For each (s,a) initialize $Q'(s,a)$ at zero

Choose current action s

Iterate infinitely{

    Choose and execute action a

    Get immediate reward r

    Choose new state s'

    Update $Q'(s,a)$ as follows:

$$Q(s,a) \leftarrow R(s) + \gamma \max_{a'} Q(s',a')$$

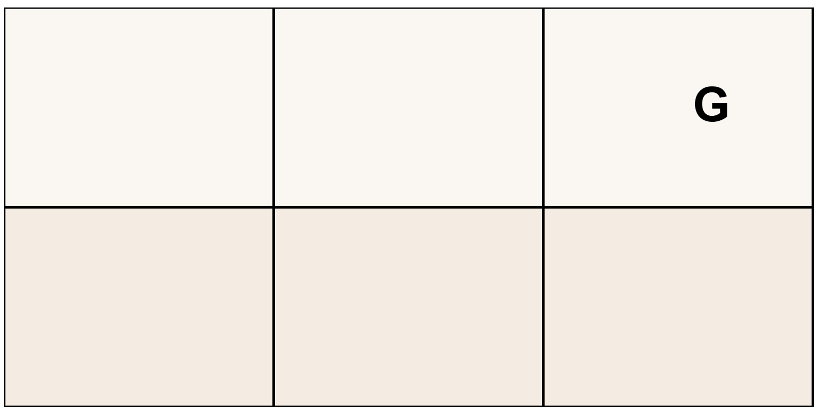
$$s \leftarrow s'$$

    }

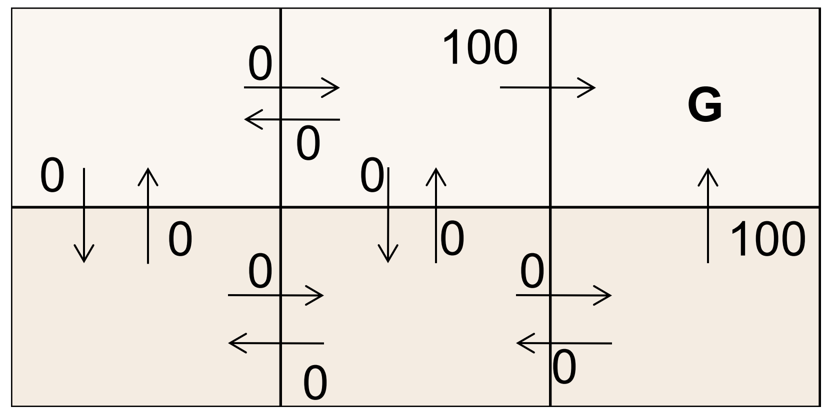
}

Example

- Initially



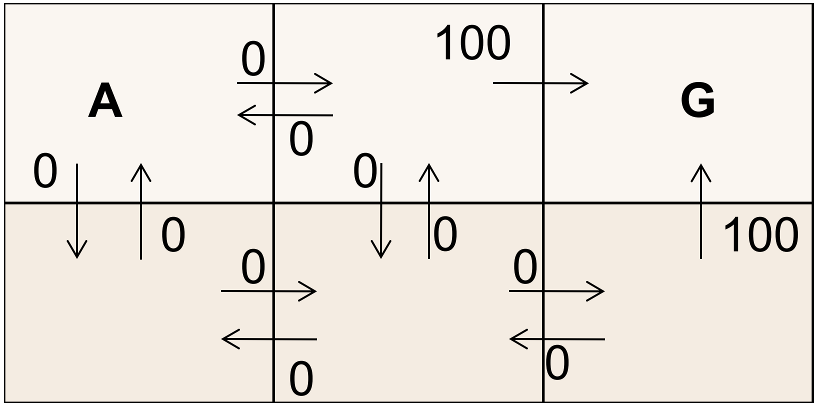
- Initialization

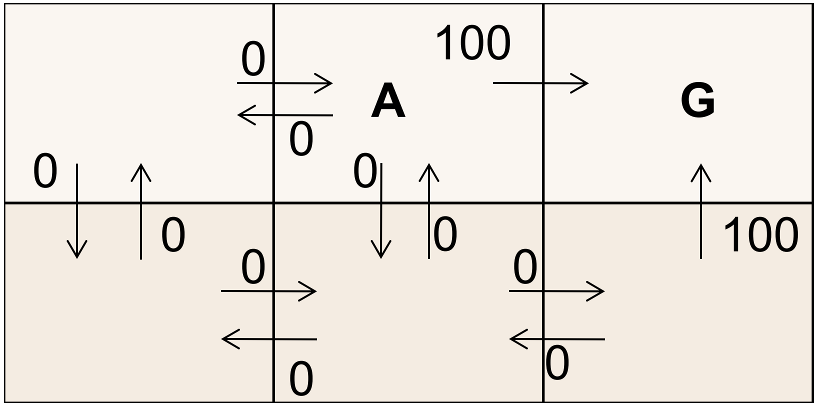


Example

- s_1

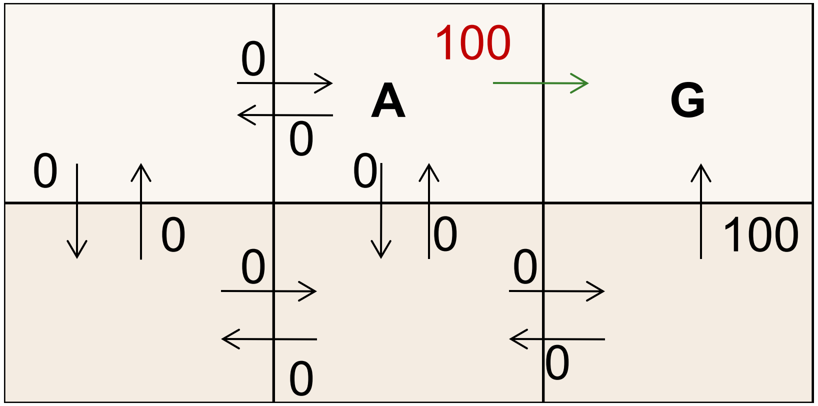
- Assume $\gamma = 0,9$
- Go right: s_2
  - Reward: 0



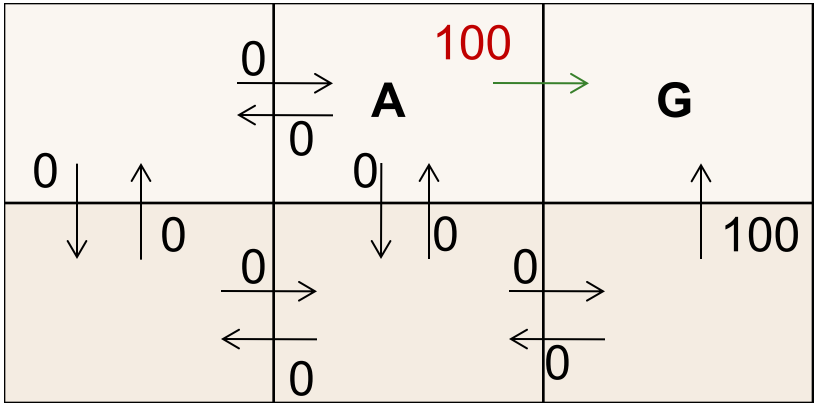


Example

- Go right
  - Reward: 100

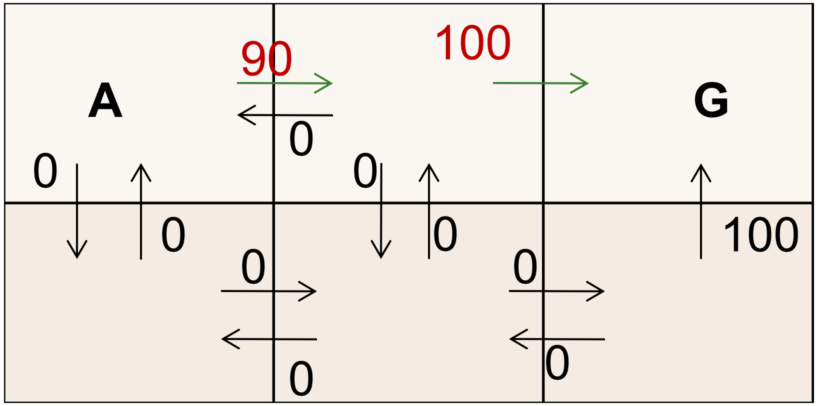


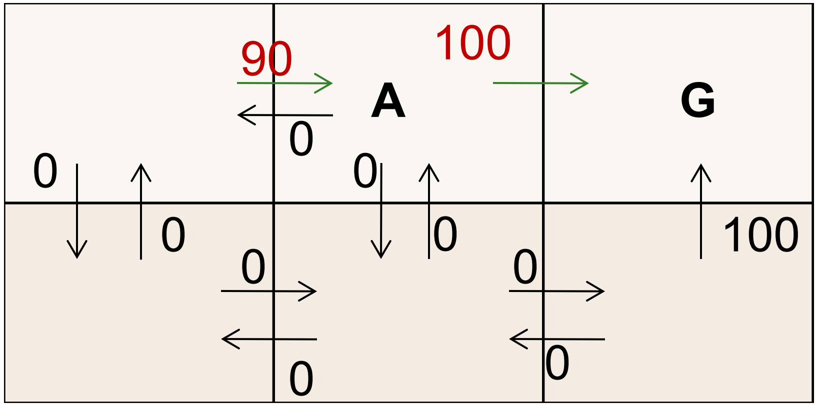
- Update s_2
  - Reward: 100



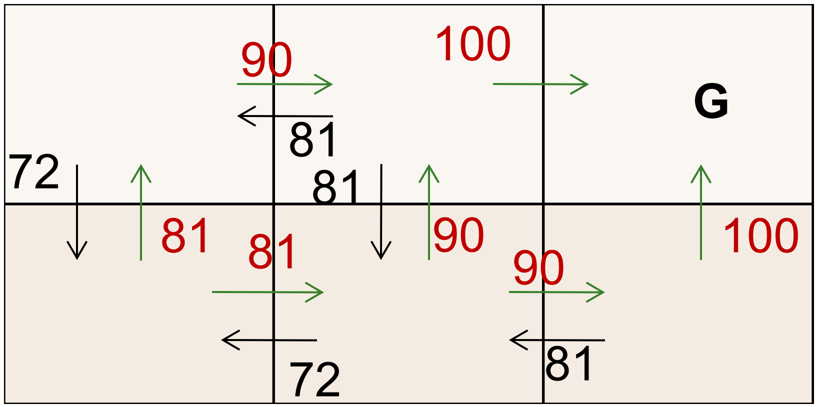
Example

- Update s_1
  - Reward: 90
- s_2



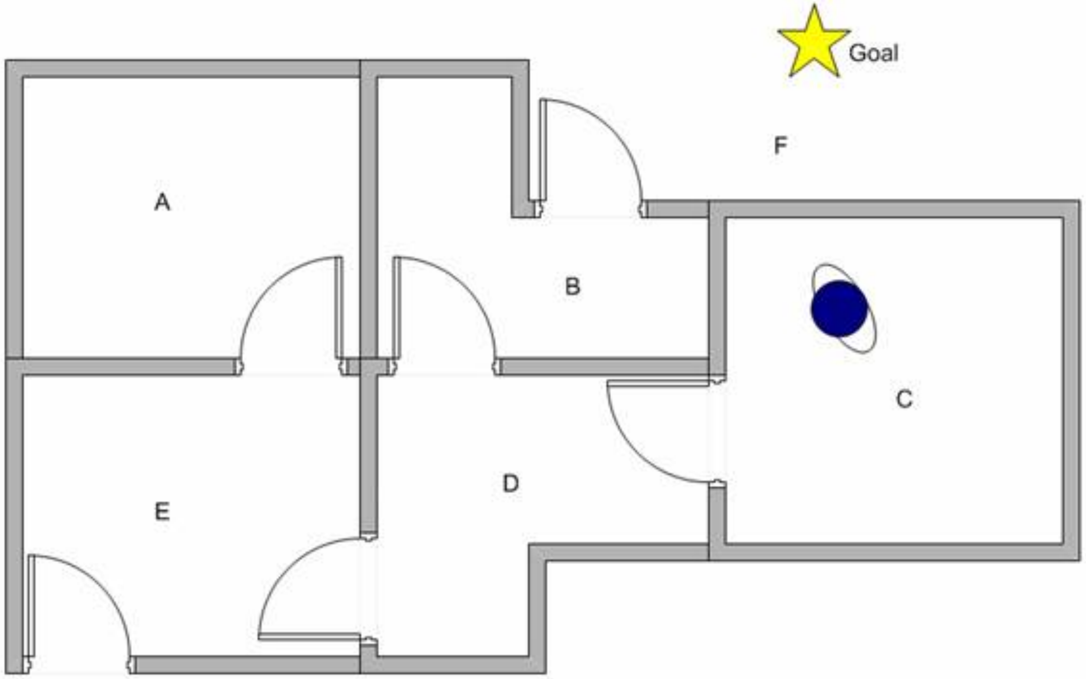


Example: result of Q-learning

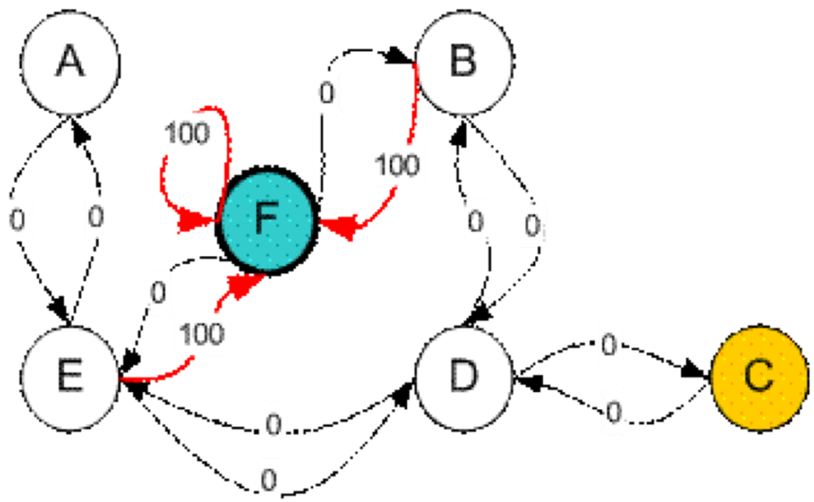


Exercice

- Agent is in room C of the building
- The goal is to get out of the building

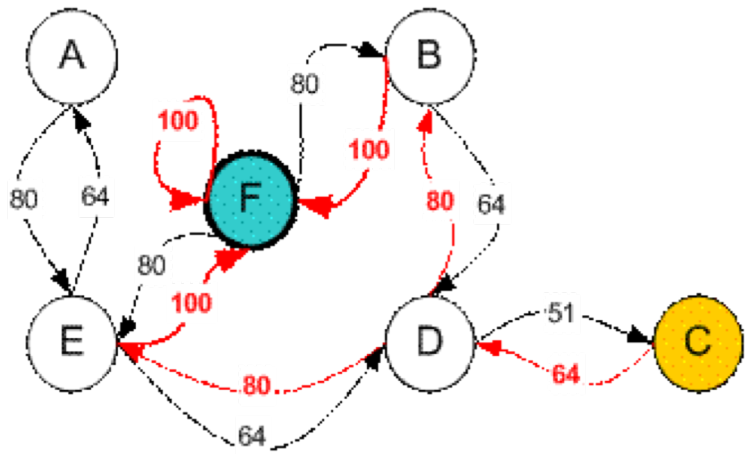


Modeling the problem



	A	B	C	D	E	F
A						
B						100
C						
D						
E						100
F						100

Result



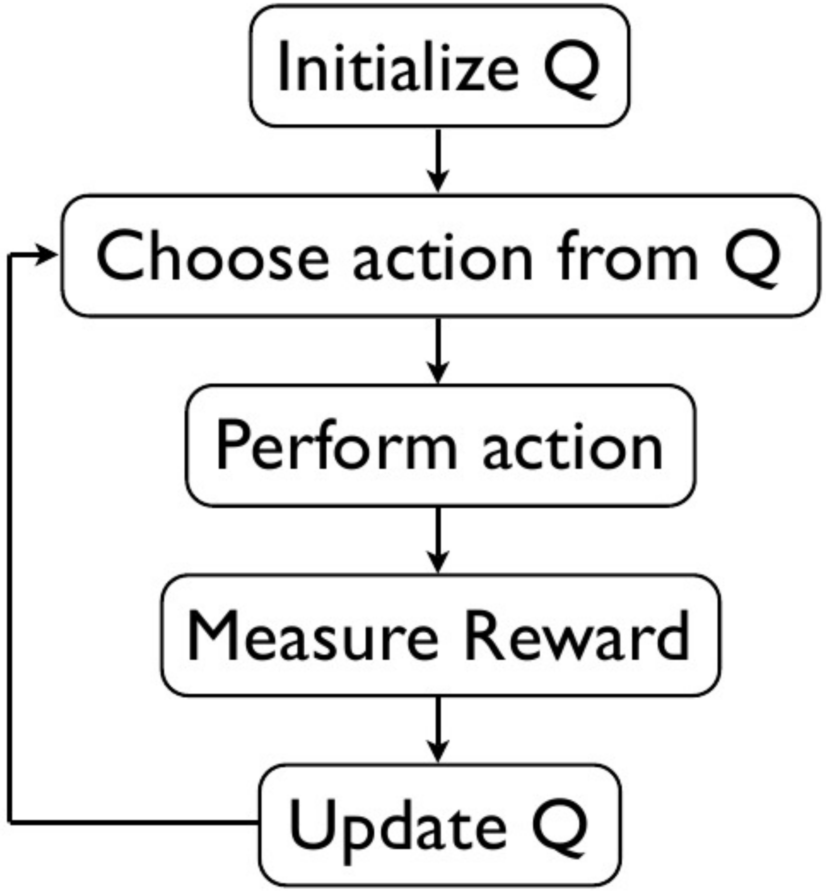
Divide all rewards by 5

$$\gamma = 0{,}8$$

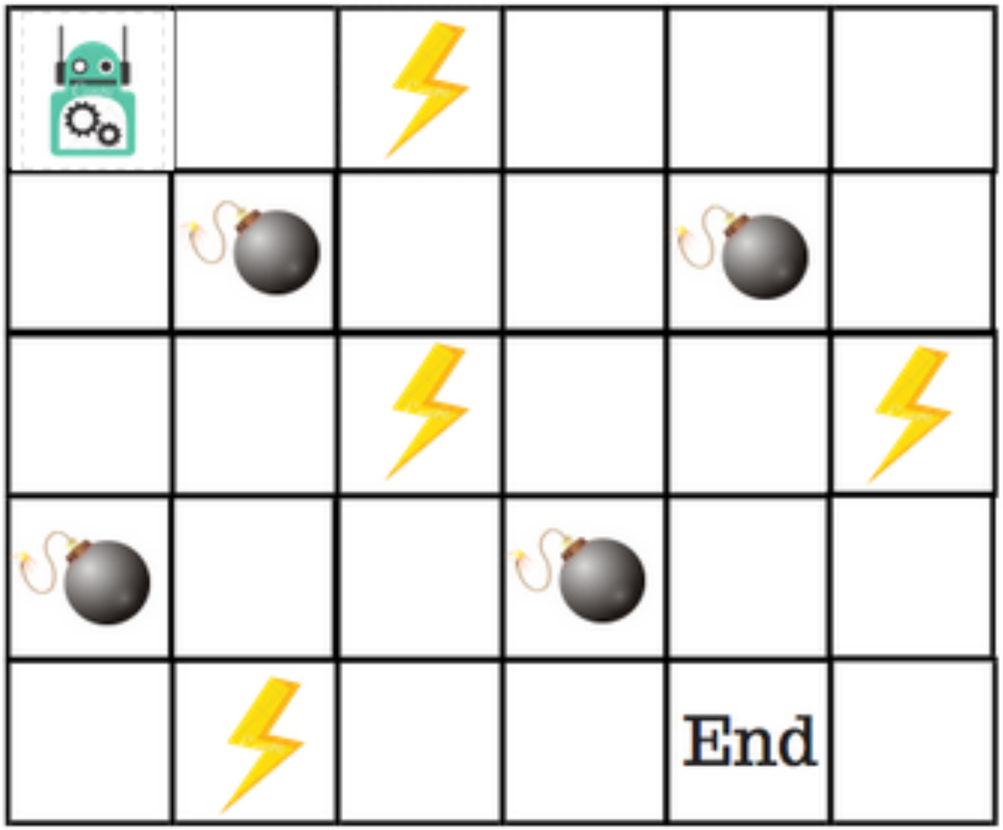
	A	B	C	D	E	F
A					400	
B				320		500
C				320		
D		400	255		400	
E	320			320		500
F		400			400	500

Result: C => D => B => F
C => D => E => F

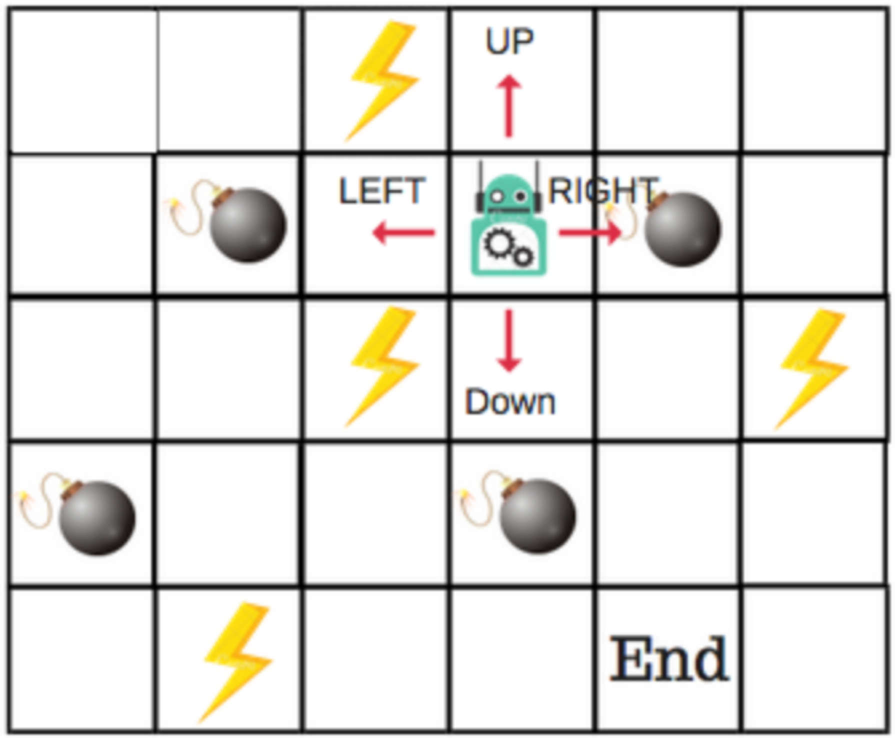
Reinforcement Learning



How do we train a robot to reach the end goal with the shortest path without stepping on a mine?



Build a "lookup table" where we calculate the maximum expected future rewards for action at each state.



- Each Q-table score will be the maximum expected future reward that the robot will get if it takes that action at that state.

- An iterative process, as we need to improve the Q-Table at each iteration..

Actions :	↑	→	↓	←
Start				
Nothing / Blank				
Power				
Mines				
END				

Update the Q(s,a) function.

$$\text{New } Q(s,a) = Q(s,a) + \alpha\,[R(s,a) + \gamma \max Q'(s',a') - Q(s,a)]$$

- New Q Value for that state and the action
- Learning Rate
- Reward for taking that action at that state
- Current Q Values
- Maximum expected future reward given the new state (s') and all possible actions at that new state.
- Discount Rate

https://en.wikipedia.org/wiki/Q-learning

- Core of Q-Learning is a simple value iteration update, using the weighted average of the old value and the new information:

$$Q^{new}(s_t, a_t) \leftarrow (1-\alpha) \cdot \underbrace{Q(s_t, a_t)}_{\text{old value}} + \underbrace{\alpha}_{\text{learning rate}} \cdot \overbrace{\bigg(\underbrace{r_t}_{\text{reward}} + \underbrace{\gamma}_{\text{discount factor}} \cdot \underbrace{\max_a Q(s_{t+1}, a)}_{\text{estimate of optimal future value}} \bigg)}^{\text{learned value}}$$

where r_t is the reward received when moving from the state s_t to the state s_{t+1}, and α is the learning rate $(0 < \alpha \leq 1)$.

Some links of interest

- https://medium.freecodecamp.org/how-to-apply-reinforcement-learning-to-real-life-planning-problems-90f8fa3dc0c5
- https://medium.freecodecamp.org/an-introduction-to-reinforcement-learning-4339519de419

Reading and Suggested Exercises

- Chapter 21
- Exercises 21.5, 21.7, 21.8